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Estimation of Errors in the Measurement of Harmonics in RHIC Arc Dipoles

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1. Introduction

A total of 298 full length dipoles of 8 cm aperture for RHIC (DRG and DR8 types) have been warm measured. Virtually all the magnets were measured twice at Northrop-Grumman (NGC) using BNL-supplied mole equipment. About 190 of these dipoles were also measured warm at BNL using another, similar mole equipment. A comparison of the results from the two warm Z-scans at NGC provides insight into the magnitude of random errors in these harmonic measurements. Similarly, a comparison of data from NGC with those at BNL provide an estimate of systematic errors due to differences in coil fabrication, calibration, and other hardware. The measurement errors are analyzed in this report in the light of these data. The errors are also estimated from first principles by considering various error modes in the construction and calibration of the measuring coils. Additional effects, such as a change in some of the harmonics after a thermal cycle or a quench, can also increase the uncertainty in the knowledge of the harmonics in a magnet, even though the measurements themselves may be very precise. Such effects are also discussed and an estimate is presented for the overall uncertainty in the measurement of harmonics in the RHIC dipoles.

2. Random Errors – Comparison of two Z-Scans at NGC

All RHIC dipoles were measured warm at NGC using a 1 m long rotating coil. Except for a few magnets (due to equipment malfunction), all magnets were also measured a second time at NGC. Each measurement was a Z-Scan with the entire length of the magnet covered by 10 axial positions of the 1 m long coil. The data from all the positions were added together to obtain the integral transfer function and the integral harmonics. A more reliable measurement of the integral transfer function was carried out using a 10 m long non-rotating coil at BNL.

The two measurements at NGC in each magnet provide a very reliable means of estimating the random errors in the measurements. Fig. 1 shows some typical histogram plots for the differences between two Z-scans. As seen from the figure, the random errors in most harmonics are quite small, typically of the order of 0.05 units (at 25 mm reference radius) or less. The reproducibility in the integral transfer function is ~0.05% (Fig. 1) which is several times larger than typical random error of ~0.01% in the measurement of body transfer function. The main source of error here is not the rotating coil system, but the axial positioning accuracy of the coil. An error of 0.05% in the integral transfer function amounts to an error of only 5 mm in the position of the coil in the ends, which is not surprising considering the fact that the coil is moved by hand using a flexible measuring tape. Table I lists the standard deviations of the differences between the two Z-scans for all harmonics. These numbers, based on data in 287 magnets, may be treated as good indicators of the random errors. It is seen from Table I that the errors are the largest for the lowest order harmonics, and become insignificant for the higher order terms. One possible reason for the larger errors for the lower order terms could be the existence of large end harmonics and larger axial variation, coupled with errors in the axial positioning of the coil.

3. Systematic Errors in the Measuring System

Systematic errors represent the differences between the measured harmonics and the true harmonics in a magnet. While it is relatively straightforward to experimentally establish the random errors, it is not so simple to estimate the systematic errors. This is because there is no way of telling what the true values of the harmonics should have been. In order to circumvent this problem, one could consider two approaches. One is to compare measurements in the same magnet with different measuring systems, and if possible, with different measuring techniques. The other approach is to analyze various possible sources of systematic errors, such as errors in the construction and calibration of the measuring coil, and then obtain estimates based on conservative (yet reasonable) values of such errors. Estimates from both of these approaches are presented in this section.

It should be pointed out here that there could be another source of systematic errors not directly related to the measuring system. These errors could arise, primarily in the skew quadrupole and the normal sextupole harmonics, due to changes in the magnet itself after a thermal cycle or a quench. In addition, there could be errors due to time dependent effects and prior excitation history of a superconducting magnet. Such errors will be most notable for the normal sextupole term, particularly at lower fields. In an overall estimate of the systematic errors in the harmonics, such effects must be included, even though the resulting errors are in no way a reflection on the capabilities of the measuring system itself.

3.1 Comparison of Z-Scans at NGC and at BNL

Once the dipoles were delivered to BNL, warm Z-scans were performed again using another mole equipment. Initially, all dipoles were measured at BNL. Once sufficient faith was established in the measuring system at NGC, only one out of two dipoles was remeasured at BNL. The sampling rate was further reduced to one out of three dipoles in order to cope with the increasing demands on the measurement resources for other magnet types for RHIC. In all, 192 full length dipoles (DRG and DR8) were warm measured at BNL.

An analysis of systematic differences between the harmonics measured at BNL and the harmonics measured at NGC can give some idea of the systematic errors. However, such an analysis suffers from two drawbacks. First, the two systems are built and calibrated in an identical way. As a result, the systematic *differences* could be small, even though the true systematic *error* could be large. Secondly, any small systematic differences may be masked by random errors. One can draw a meaningful conclusion only in instances where the observed systematic differences are large compared to the random errors. These drawbacks not withstanding, some typical results of a comparison between the BNL and NGC data are shown in Fig. 2. A summary of systematic errors based on this comparison is given in Table II. Except for the very high order 30-pole terms, practically no systematic differences are seen between the two measurements. The 30-pole term is particularly sensitive to systematic errors for two reasons – (a) any systematic errors (expressed as a percentage error) due to coil calibration or construction errors generally increase with harmonic number and (b) the magnitude of the 30-pole term in the RHIC arc dipoles is rather large, 0.78 unit. The systematic difference (average difference in 186 magnets) between the BNL and NGC values for the amplitude of the 30-pole term is 0.019 unit. Although small in magnitude, this difference is almost 10 times the random error (see Table I) and amounts to an error of 2.4%, which is not inconsistent with independent estimates of systematic errors based on reasonable construction errors (see Sec. 3.2). A similar comment applies to other high order allowed terms, such as the normal 22-pole and 26-pole terms. The systematic differences for all the other terms are less than, or comparable to the random errors. An exception is the unallowed 28-pole term for which the systematic difference is larger than the random error. This systematic difference in 28-pole term is perhaps a result of small errors in centering. An error of only ~10 μm in the centering calculation will create a feed down 28-pole of ~0.005 unit from 0.78 unit of 30-pole. Another source of error is the use of harmonics up to only the 30-pole in calculating the centered harmonics. Thus, the second and higher order centering corrections to 28-pole term are not available. Such correction terms are crucial for the accuracy of very high order harmonics. It would not be surprising if these corrections amount to ~0.005 unit.

3.2 Systematic errors due to coil construction and calibration errors

The primary source of systematic measurement errors stems from imperfections in the construction and calibration of the measuring coils. Even though the measuring coils are built with utmost care and precision, there are bound to be deviations from an ideal geometry. These deviations give rise to errors in the determination of harmonics. Various construction and calibration errors for measuring coils are discussed in detail in a recent review [1]. The error modes are briefly reviewed here and estimates of systematic errors from each of the error modes are given in Table III.

3.2.1 Systematic Error in Radius

This is an error in the calibration of the measuring coil. If ΔR is the calibration error in a coil of radius R, then the error $\Delta C(n)$ in the amplitude, $C(n)$, of the $2n$-pole term is

$$\frac{\Delta C(n)}{C(n)} = n\left(\frac{\Delta R}{R}\right) \tag{1}$$

The radii of RHIC measuring coils are estimated in various ways. Discrepancies between different methods suggest an uncertainty $\Delta R / R$ ~ 0.001 corresponding to a ΔR of ~25 μm in a coil of 25 mm radius. The resulting errors are given in Table III. In addition to the errors in the harmonics, this error is the main source of systematic error in the transfer function in the straight section of the magnet.

3.2.2 Systematic Error in Opening Angle of the Tangential Coil

This is also a calibration error. If ε_Δ is the error in the opening angle Δ of a tangential coil, then the error in the amplitude of the 2*n*-pole term is given by

$$\frac{\Delta C(n)}{C(n)} = \left(\frac{n}{2}\right)\cot\left(\frac{n\Delta}{2}\right)\varepsilon_\Delta \tag{2}$$

For the RHIC coils, $\Delta \approx 15$ degrees and is estimated from an elaborate calibration procedure in dipole, quadrupole and sextupole fields [1]. Table III gives the calculated errors for a rather conservative error of 1 mrad in estimating Δ.

3.2.3 Random Variation in the Coil Radius

If there is a variation in the radius of the measuring coil along its length, there is an error in calculating the harmonics. This is a coil construction error and its effect can not be removed by a good calibration. If σ_R is the RMS variation in the radius, the error is

$$\frac{\Delta C(n)}{C(n)} \approx \frac{n(n-1)}{2}\left(\frac{\sigma_R}{R}\right)^2 \tag{3}$$

The random variation is expected to be larger than the error in estimating the mean value. Based on typical variations seen in the measured groove depths in RHIC coil forms, it is expected that σ_R/R is no more than 0.003. The errors for this are given in Table III.

3.2.4 Random Variation in the Angular Position of the Grooves

The windings in the measuring coils are placed into machined grooves in the coil form. The azimuthal location of the groove varies along the length due to construction errors. If σ_δ is the RMS variation in the angular position of the winding, the harmonic errors are

$$\frac{\Delta C(n)}{C(n)} \approx \frac{n^2}{2}\sigma_\delta^2 \tag{4}$$

Inspection data on various RHIC coil forms, as well as calibration of angles of short sections of long measuring coils shows that the angular position of a groove may vary by as much as 4 mrad from the ideal value. Table III lists the calculated errors for $\sigma_\delta = 0.004$.

3.2.5 Random Variation in Opening Angle of the Tangential Winding

As the grooves are located at different angular positions along the length of the measuring coil, the opening angle of the tangential winding will also vary along the length. If σ_Δ is the RMS variation in the opening angle, the errors are given by

$$\frac{\Delta C(n)}{C(n)} \approx \frac{n^2}{8}\sigma_\Delta^2 \tag{5}$$

Table III lists the calculated errors for σ_Δ = 0.004 radian, which is the same as σ_δ used in the previous subsection. A similar value is justified because the same type of construction error is responsible for both of these.

3.2.6 An Offset in Rotation Axis

If the rotation axis of the measuring coil is offset from the geometric axis of the windings by a distance r_0, then the measured harmonics will be in error by an amount given by

$$\frac{\Delta C(n)}{C(n)} \approx n\left(\frac{r_0}{R}\right)\frac{\sin\left[\frac{(n-1)\Delta}{2}\right]}{\sin\left[\frac{n\Delta}{2}\right]} \tag{6}$$

where R is the radius of the measuring coil. Strictly speaking, an offset in rotation axis is a fixed construction error which leads to a change in the effective radii and the opening angles of the windings. In principle, the effect can be largely compensated by a proper calibration. Thus, in a sense, this error is implicitly included in the other calibration errors already discussed. Nevertheless, in order to obtain a conservative estimate, Table III includes independent estimates of errors for a 0.002″ offset in a 1″ radius measuring coil.

3.2.7 Total Systematic Errors due to Coil Construction/Calibration

Assuming that all the error modes add up, one can obtain an estimate of the maximum systematic error. This total error, expressed as a percentage value, is shown in Table III under the heading “Total error possible”. These percentage errors are multiplied by the largest measured value of each harmonic in the entire production of DRG/DR8 magnets to obtain the maximum systematic errors in units, listed in the last two columns of Table III. These numbers represent the best estimates of systematic errors in the measurement process itself. One must add the uncertainties due to changes in the magnet itself to obtain the final measurement errors.

4. Systematic Errors due to Changes in the Magnet

In addition to errors due to the measuring system, there may be errors due to changes in the magnet itself. Two most important effects in this category are the time dependence of harmonics due to decay of persistent currents and changes in harmonics after a thermal cycle and/or quench.

4.1 Time Dependence of Harmonics

In the superconducting state, time decay of persistent currents can lead to a time dependence of certain allowed harmonics. This problem is most severe for the normal sextupole at injection field. At full field, the contribution from persistent currents is significantly reduced. The harmonics in all the RHIC dipoles tested cold were measured in two ways. In the "Z-scan" method, the magnet is powered at a fixed current and measurements are carried out at different axial locations by moving the mole. The magnet stays at a fixed current for a long time (~30 minutes) in this method. In the "DC loop" method, the mole is held fixed at one axial location and the current is ramped up in small steps. The harmonics are measured at each current step after a wait of about 15 seconds. The magnet was cycled from 0A to 5000A and back to 0A before the start of any run. In some magnets, the time dependence was specifically studied by ramping the magnet to 660A (injection field) or 5000A (storage field), and then measuring the harmonics roughly every 10 seconds for about 10 minutes. Fig. 3 shows the time dependence of the normal sextupole harmonic measured at an excitation of 5kA (full field) in the magnet DRG102. The values measured at the same location in a Z-scan (solid line) and in the upward current ramp of a DC loop (dashed line) are also shown for comparison. It is seen that the normal sextupole decays with time over a period of several minutes. The uncertainty in the normal sextupole due to this effect is of the order of 0.1 unit at full field. This uncertainty is several times more at injection field (660 A), as seen from Fig. 4.

4.2 Changes in Harmonics after Thermal Cycle and/or Quench

It has been observed in some RHIC magnets that the harmonics change after a thermal cycle or a quench. This effect has been studied [2] in the 13 cm aperture quadrupoles and 100 mm aperture dipoles for the RHIC insertion region. Only limited data are available on this effect in the RHIC arc dipoles. As an example, Fig. 5 shows the normal sextupole and skew quadrupole harmonics at 5kA in the 8 straight section locations in three Z-scans in the magnet DRG101. The first two Z-scans (crosses and triangles) were done in the same cool down cycle. The excellent reproducibility of measurements is quite evident. The third scan (diamonds) was carried out after a thermal cycle. A systematic shift, much larger than the random errors, is seen in both the harmonics. Although some change is seen in several harmonics, the skew quadrupole and the normal sextupole seem to be the most affected by thermal cycle and quench in dipoles.

5. Overall Estimate of Errors

In order to arrive at an overall estimate of errors, we combine all the systematic and random errors discussed in this note. Table IV summarizes all the sources of errors considered in this note. The total expected error is a sum of all the individual errors. The maximum systematic error due to coil construction or calibration is taken from Table III. The random errors are based on comparison of two Z-scans at NGC, and are taken from Table I. The "suggested values" in the last column are obtained by upward rounding of the total errors. A minimum error of 0.02 unit is assumed to account for effects that may not have been considered in this note. Thus, all values below 0.02 unit are rounded up to 0.02 unit. Similarly, all values between 0.02 unit and 0.05 unit are rounded up to 0.05 unit. Error values higher than 0.05 unit are rounded up to the nearest 0.1 unit. It is believed that these suggested values represent a fairly close estimate of the measurement errors in the RHIC dipoles.

References

[1] Animesh K. Jain, *Harmonic Coils*, Proc. CERN Accelerator School on Measurement and Alignment of Accelerator and Detector Magnets, April 11-17, 1997, Anacapri, Italy, to be published.

[2] R. Gupta, A. Jain, J. Muratore, P. Wanderer, E. Willen, and C. Wyss, *Change in Field Harmonics After Quench and Thermal Cycles in Superconducting Magnets*, Proc. 1997 Particle Accelerator Conference, 1997.

Table I

Random errors based on comparison of harmonics from two Z-scans (287 magnets)

Harmonic	Std. Dev. of difference in harmonics (units at 25 mm)	
	Normal	**Skew**
Quadrupole	0.061	0.043
Sextupole	0.033	0.015
Octupole	0.012	0.010
Decapole	0.004	0.005
Dodecapole	0.003	0.004
14-pole	0.002	0.002
16-pole	0.001	0.002
18-pole	0.001	0.001
20-pole	0.001	0.001
22-pole	0.001	0.001
24-pole	0.001	0.001
26-pole	0.001	0.001
28-pole	0.002	0.002
30-pole	0.002	0.002

Table II

Systematic errors based on comparison of harmonics from Z-scans at BNL and NGC (186 magnets)

Harmonic	Average difference in harmonics (units at 25 mm)	
	Normal	**Skew**
Quadrupole	-0.031	0.023
Sextupole	0.001	-0.001
Octupole	-0.008	0.001
Decapole	0.001	-0.003
Dodecapole	-0.003	-0.003
14-pole	0.002	-0.003
16-pole	0.000	-0.001
18-pole	0.002	-0.001
20-pole	-0.002	0.000
22-pole	-0.009	0.000
24-pole	-0.001	0.000
26-pole	-0.005	-0.001
28-pole	-0.004	0.006
30-pole	0.019	0.009

Table III

Estimation of Systematic Errors due to Various Errors in the Construction and Calibration of Coils

Harmonic Number (n = 1 denotes dipole)	Systematic error in radius	Systematic Error in opening angle	Random variation in radius	Random variation in angular position	Random variation in opening angle	Offset in rotation axis	Total systematic error possible	Maximum value of harmonic in DRG/DR8 magnets (units)		Maximum systematic error due to coil construction/ calibration (units)	
	$\Delta R/R$	ε_Δ (rad)	σ_R/R	σ_δ (rad)	σ_Δ (rad)	r_0/R		Normal	Skew	Normal	Skew
	0.001	0.001	0.003	0.004	0.004	0.002					
1	0.10%	0.38%	0.00%	0.00%	0.00%	0.00%	0.48%	--	--	--	--
2	0.20%	0.37%	0.00%	0.00%	0.00%	0.20%	0.78%	1.380	5.881	0.011	0.046
3	0.30%	0.36%	0.00%	0.01%	0.00%	0.41%	1.08%	7.866	1.729	0.085	0.019
4	0.40%	0.35%	0.01%	0.01%	0.00%	0.61%	1.38%	0.293	1.399	0.004	0.019
5	0.50%	0.33%	0.01%	0.02%	0.01%	0.82%	1.68%	1.334	0.335	0.022	0.006
6	0.60%	0.30%	0.01%	0.03%	0.01%	1.03%	1.98%	0.107	0.516	0.002	0.010
7	0.70%	0.27%	0.02%	0.04%	0.01%	1.25%	2.28%	0.527	0.191	0.012	0.004
8	0.80%	0.23%	0.03%	0.05%	0.01%	1.47%	2.59%	0.042	0.143	0.001	0.004
9	0.90%	0.19%	0.03%	0.06%	0.02%	1.69%	2.89%	0.316	0.045	0.009	0.001
10	1.00%	0.13%	0.04%	0.08%	0.02%	1.91%	3.19%	0.019	0.032	0.001	0.001
11	1.10%	0.07%	0.05%	0.10%	0.02%	2.14%	3.49%	0.580	0.015	0.020	0.001
12	1.20%	0.00%	0.06%	0.12%	0.03%	2.38%	3.78%	0.008	0.020	0.000	0.001
13	1.30%	0.09%	0.07%	0.14%	0.03%	2.62%	4.25%	0.214	0.028	0.009	0.001
14	1.40%	0.19%	0.08%	0.16%	0.04%	2.87%	4.74%	0.062	0.046	0.003	0.002
15	1.50%	0.31%	0.09%	0.18%	0.05%	3.14%	5.27%	0.777	0.080	0.041	0.004

Table IV

Summary of various contributions to measurement errors. The normal and skew harmonics are indicated using the US notation (b_1 = normal quadrupole, etc.)

Harmonic	Maximum error due to meas. coil construction/ calibration (units)	Effect of thermal cycle and/or quench (units)	Effect of time dependence, at 5kA (units)	Random error in measure-ment (units)	Total expected error (units)	Suggested value of total measurement uncertainty (units)
b_1	0.011	0.006	0.0	0.061	0.078	0.10
b_2	0.085	0.203	0.1	0.033	0.420	0.50
b_3	0.004	0.009	0.0	0.012	0.026	0.05
b_4	0.022	0.044	0.0	0.004	0.071	0.10
b_5	0.002	0.012	0.0	0.003	0.016	0.02
b_6	0.012	0.005	0.0	0.002	0.019	0.02
b_7	0.001	0.000	0.0	0.001	0.003	0.02
b_8	0.009	0.003	0.0	0.001	0.013	0.02
b_9	0.001	0.004	0.0	0.001	0.006	0.02
b_{10}	0.020	0.001	0.0	0.001	0.022	0.05
b_{11}	0.000	0.002	0.0	0.001	0.003	0.02
b_{12}	0.009	0.002	0.0	0.001	0.012	0.02
b_{13}	0.003	0.002	0.0	0.002	0.006	0.02
b_{14}	0.041	0.004	0.0	0.002	0.047	0.05
a_1	0.046	0.388	0.0	0.043	0.477	0.50
a_2	0.019	0.000	0.0	0.015	0.034	0.05
a_3	0.019	0.027	0.0	0.010	0.056	0.10
a_4	0.006	0.002	0.0	0.005	0.013	0.02
a_5	0.010	0.009	0.0	0.004	0.023	0.05
a_6	0.004	0.000	0.0	0.002	0.006	0.02
a_7	0.004	0.001	0.0	0.002	0.006	0.02
a_8	0.001	0.006	0.0	0.001	0.008	0.02
a_9	0.001	0.001	0.0	0.001	0.003	0.02
a_{10}	0.001	0.001	0.0	0.001	0.003	0.02
a_{11}	0.001	0.001	0.0	0.001	0.003	0.02
a_{12}	0.001	0.008	0.0	0.001	0.010	0.02
a_{13}	0.002	0.001	0.0	0.002	0.005	0.02
a_{14}	0.004	0.008	0.0	0.002	0.014	0.02

Differences between two warm Z-Scans at Northrop-Grumman in RHIC arc dipoles

(Integral harmonics at 25 mm reference radius)

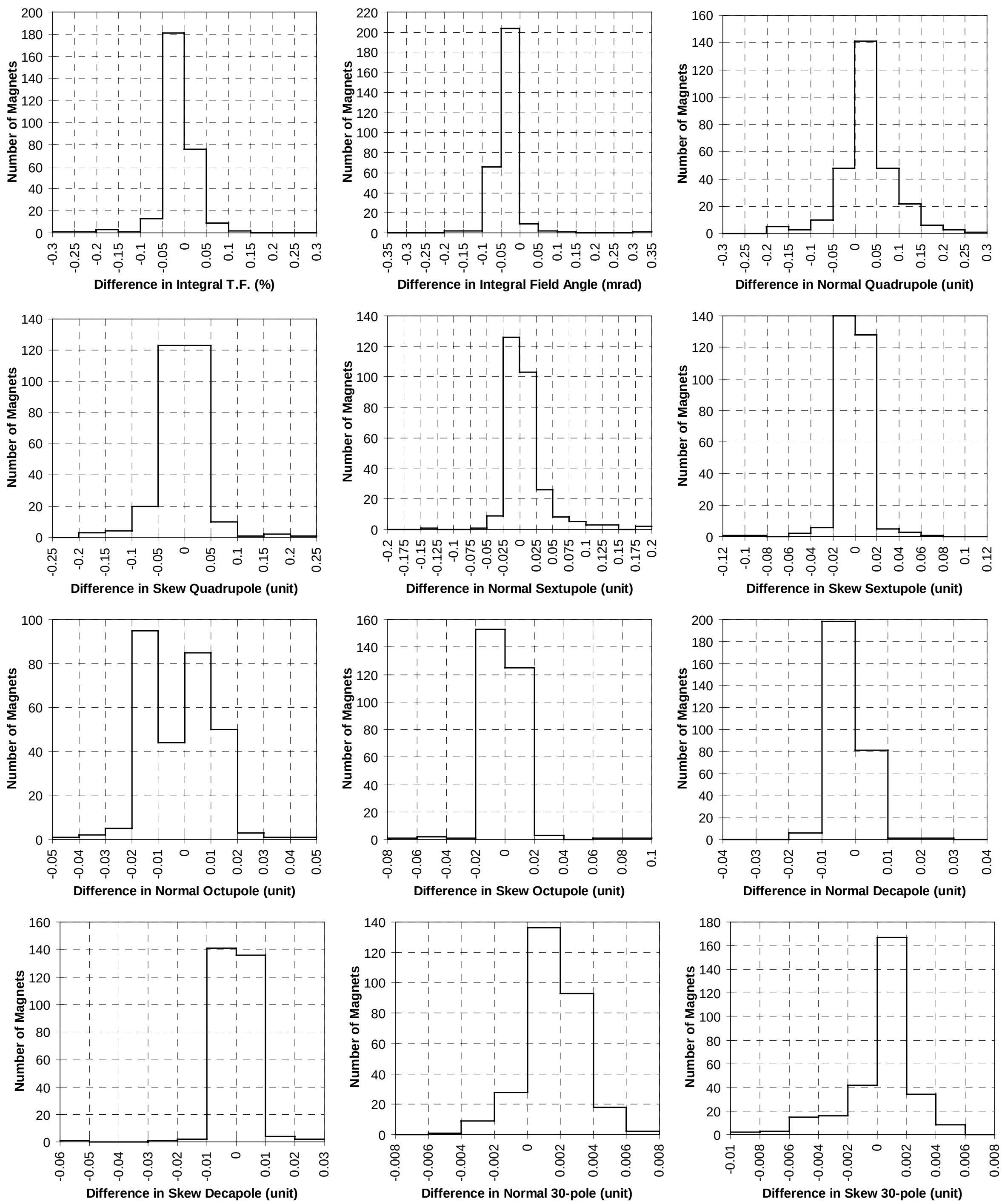

Fig. 1 Histograms showing the distribution of random errors in some of the harmonics.

Differences between Warm Z-Scans at NGC and BNL in RHIC arc dipoles
(Integral harmonics at 25 mm reference radius)

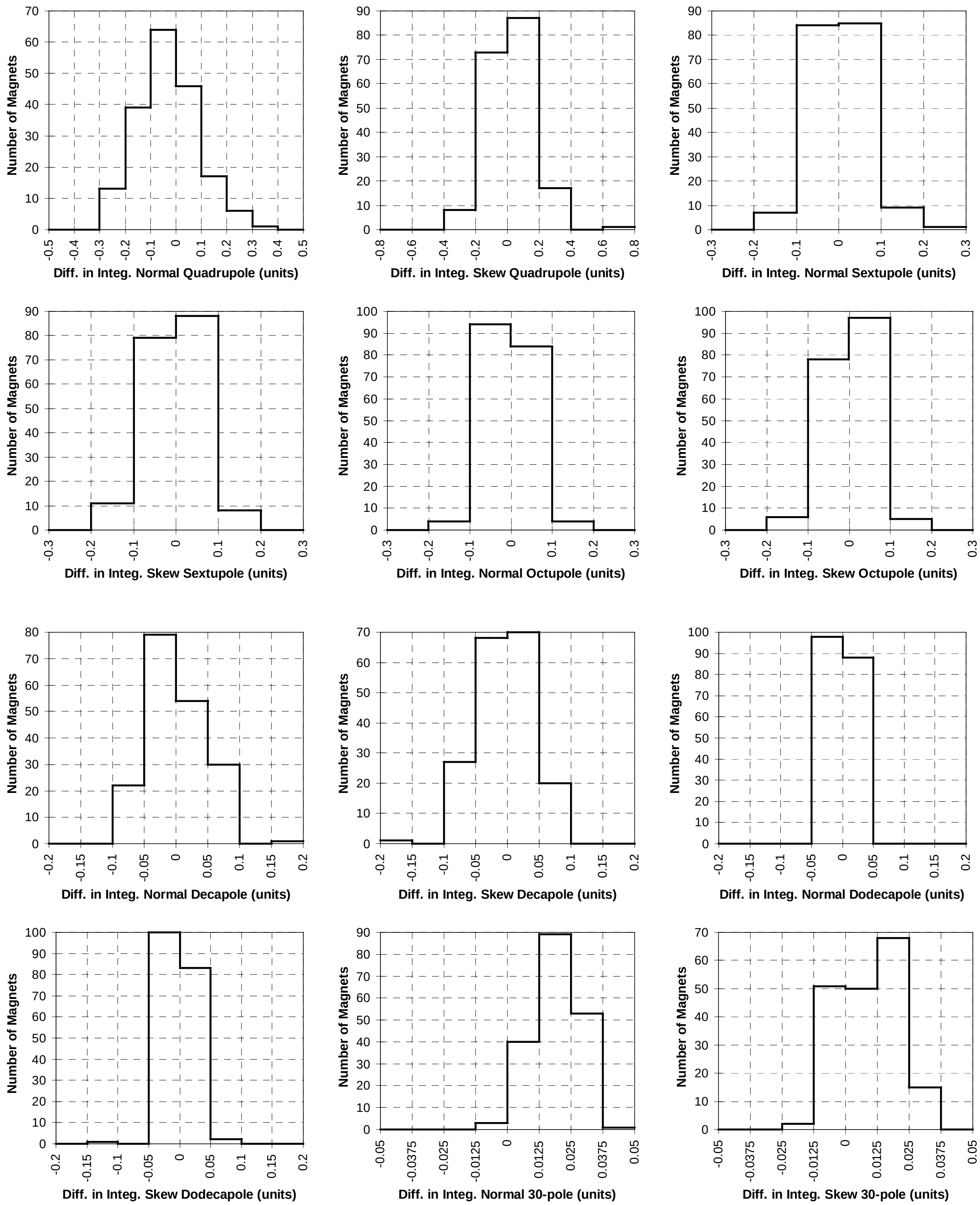

Fig. 2 Histograms showing the distribution of differences between BNL and NGC data.

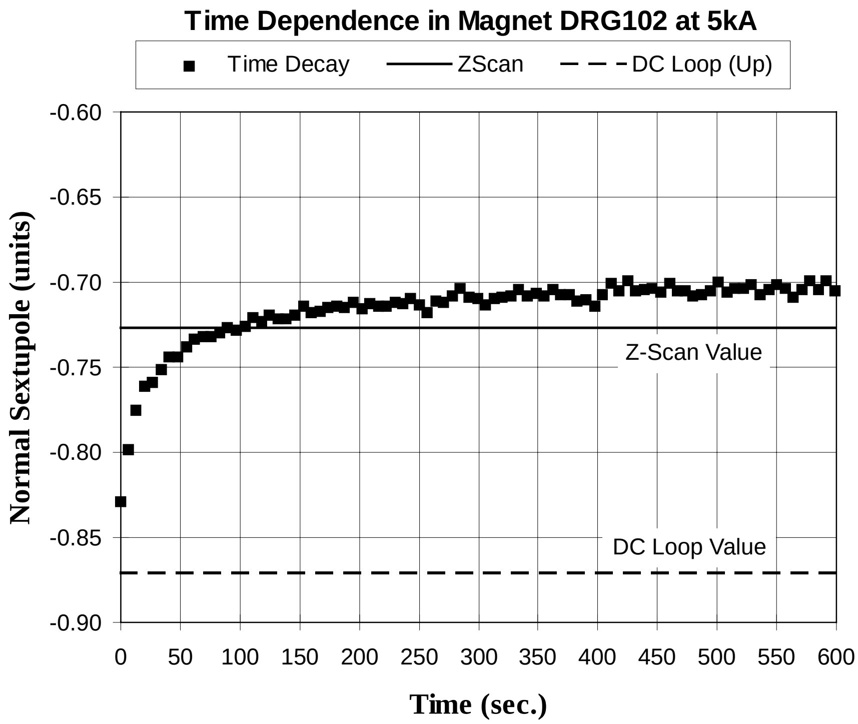

Fig. 3 Time dependence of normal sextupole at full field (5kA)

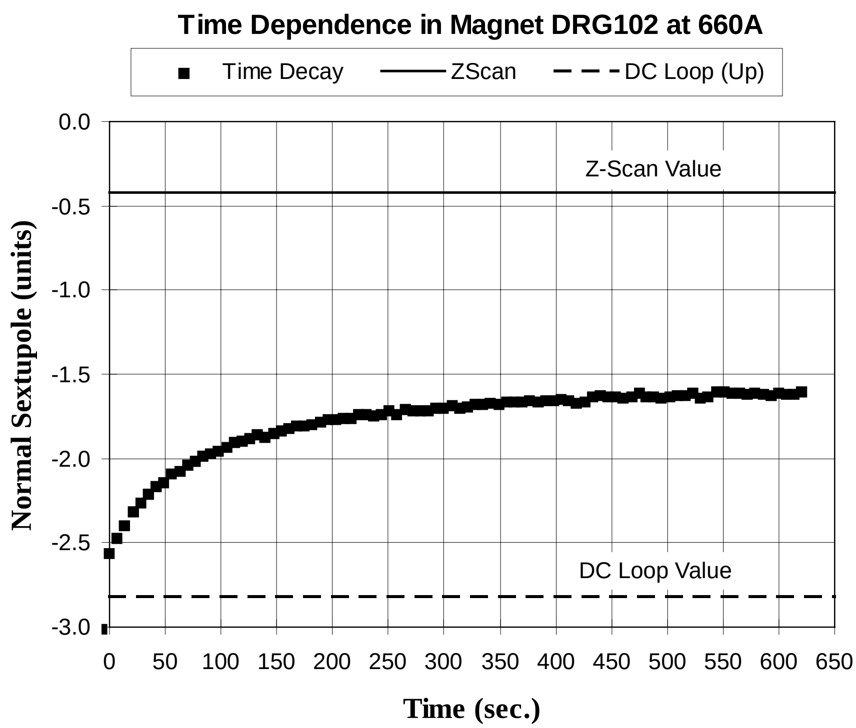

Fig. 4 Time dependence of normal sextupole at injection field (660 A)

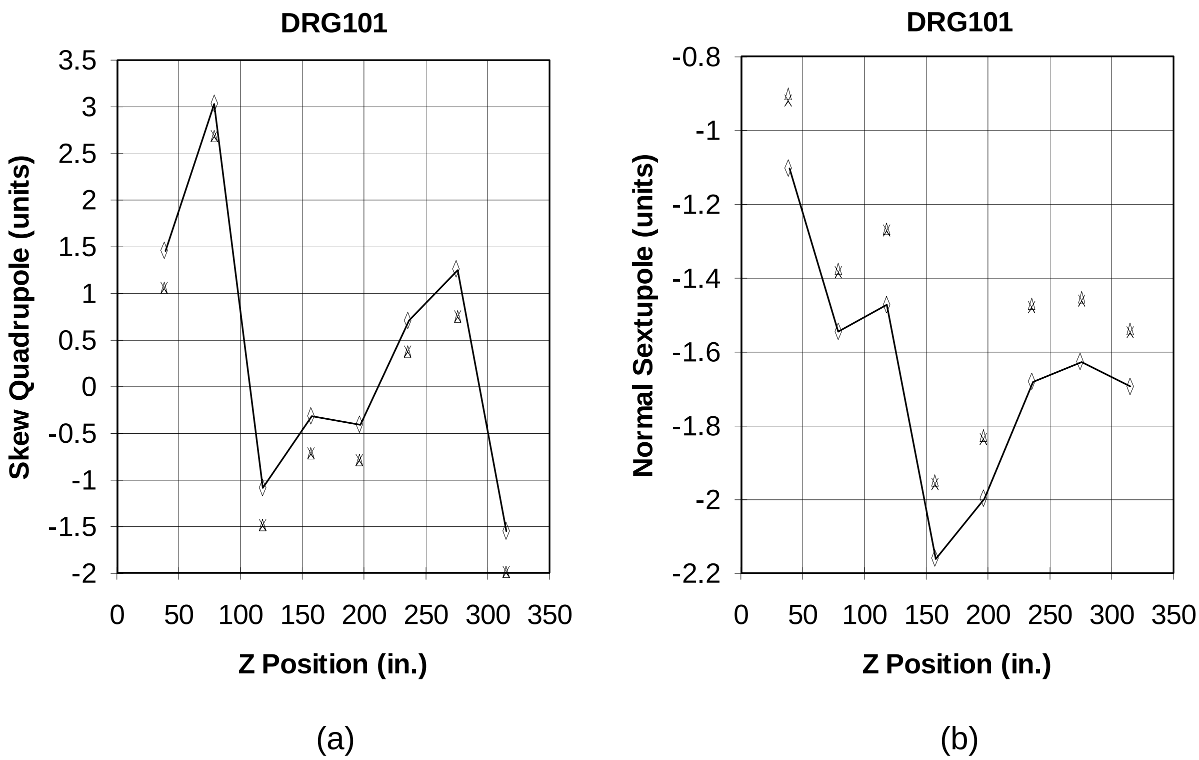

Fig. 5 Harmonics measured at the 8 central locations in DRG101 in two Z-scans before a thermal cycle (crosses and triangles), and a Z-scan after a thermal cycle (diamonds).